

# Galileo satellite constellation and extensions to General Relativity

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We consider the impact of some known extensions of General Relativity in observables that will be available with the Galileo positioning systems, and draw conclusions as to the possibility of measuring them. We specifically address the effects of the presence of a cosmological constant, a Yukawa-like addition to the Newtonian potential, and the existence of an extra, constant acceleration. We also consider the phenomenological impact of a broad class of metric theories, which can be classified through the parameterised Post-Newtonian formalism.

## I. INTRODUCTION

The Galileo positioning system poses a great opportunity, not only for the improvement and development of new applications in navigation monitoring and related topics, but also possibly for fundamental research in physics. Indeed, together with the already deployed american and russian counterparts, the Global Positioning System (GPS) and Glonass, satellite navigation may be considered the first practical application where relativistic effects are taken into account, not from an experimental point of view, but as a regular engineering constraint on the overall design requirements. Indeed, effects arising from special and General Relativity (GR) – gravitational blueshift, time dilation and Sagnac effect – may account to as much as  $\sim 40 \mu s/day$ , which is many orders of magnitude above the accuracy of the onboard clock deployed in these systems. Moreover, the gravitational Doppler effect, of the order of  $V_N/c^2 \sim 10^{-10}$  (where  $V_N = GM_E/R_E$  is the Newtonian potential,  $G$  is Newton's constant,  $M_E \approx 6.0 \times 10^{24} kg$  is the Earth's mass,  $R_E \approx 6.4 \times 10^6 m$  is its radius and  $c$  is the speed of light) falls within the  $10^{-12}$  frequency accuracy of current space-certified clocks, and must therefore be taken into account: in GPS, this is done by imposing an offset in the onboard clock frequency, while in Galileo this correction should be corrected by the receiver. For further details, the reader is directed to Refs. [1–4] and references within.

This said, it is not clear as to what extent the accuracy of the Galileo positioning system may be improved – which is designed to offer pinpoint localisation within an error margin of 1  $m$ , against the 10  $m$  margin of previous the GPS system – so to provide clues to the nature of models beyond the current GR scenario. In this study, we aim at establishing some bounds on the detectability of commonly considered extensions to GR [5]. This paper is organised as follows: firstly, we assess the main relativistic effects that are present in the Galileo system. We proceed and consider the possibility of measuring a variety of extensions of GR and conclusions are then drawn.

## II. MAIN RELATIVISTIC EFFECTS

### A. Frame of reference

Assuming that time-dependent effects are of cosmological origin, and hence of order  $H_0^{-1}$ , where  $H_0$  is Hubble's constant, one may discard these as too small within the timeframe of interest; hence, one assumes a static, spherically symmetric scenario, posited by the standard Schwarzschild metric. In isotropic form, this is given by the line element

$$ds^2 = - \left( 1 + \frac{2V}{c^2} \right) (c dt)^2 + \frac{1}{1 + \frac{2V}{c^2}} dV \cong - \left( 1 + \frac{2V}{c^2} \right) (c dt)^2 + \left( 1 - \frac{2V}{c^2} \right) dV, \quad (1)$$

where  $dV = dr^2 + d\Omega^2$  is the volume element, and  $V$  is the gravitational potential. In the standard GR scenario, the latter coincides with the Newtonian potential  $V = V_N = -GM_E/r(1 + \sum_{i=1}^n J_n)$ , where the  $J_n$  multipoles account for the effect of geographic perturbations and density profiles.

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However, one must introduce the rotation of the Earth with respect to this fixed-axis reference frame, with angular velocity  $\omega = 7.29 \times 10^{-5} \text{ rad/s}$ ; by doing a coordinate shift  $t' = t$ ,  $r' = r$ ,  $\theta' = \theta$  and  $\phi' = \phi - \omega t'$ , one gets the Langevin metric, given by the line element

$$ds^2 = - \left[ 1 + \frac{2V}{c^2} - \left( \frac{\omega r \sin \theta}{c} \right)^2 \right] (c dt)^2 + 2\omega r^2 \sin^2 \theta d\phi dt + \left( 1 + \frac{2V}{c^2} \right) dV, \quad (2)$$

where, for simplicity, primes were dropped. Besides from a non-diagonal element, one obtains an addition to the gravitational potential, which could be viewed as a centrifugal contribution due to the rotation of the reference frame. One can then define an effective potential  $\Phi = 2V - (\omega r \sin \theta)^2$ ; the parameterization of the Earth's geoid is obtained by taking the multipole expansion of  $V$  up to the desired order and finding the equipotential lines  $\Phi = \Phi_0$  (the latter being the value of  $\Phi$  at the equator), and solving for  $r(\theta, \phi)$ .

In the above line elements, the coordinate time coincides with the proper time of an observer at infinity. However, since one wishes to evaluate the ground to orbit clock synchronisation, it is advantageous to rewrite the metric in terms of a rescaled time coordinate, which coincides with the proper time of clocks at rest on the surface of the Earth; this is best implemented by resorting to the above-mentioned geoid, since its definition as an equipotential surface  $\Phi = \Phi_0$  indicates that all clocks at rest in it beat at the same rate; hence, rescaling the time coordinate according to  $t \rightarrow (1 + \Phi_0/c^2)t$ , one gets the metric given by the line element

$$ds^2 = - \left[ 1 + \frac{2(\Phi - \Phi_0)}{c^2} \right] (c dt)^2 + 2\omega r^2 \sin^2 \theta d\phi dt + \left( 1 - \frac{2V}{c^2} \right) d\Omega. \quad (3)$$

Finally, if one reassumes a non-rotating frame, the metric is given by the line element

$$ds^2 = - \left[ 1 + \frac{2(V - \Phi_0)}{c^2} \right] (c dt)^2 + \left( 1 - \frac{2V}{c^2} \right) d\Omega. \quad (4)$$

## B. Constant and periodic clock deviation

One may now consider the difference between the time elapsed on the ground and the satellite clock; keeping only terms of order  $c^{-2}$ , one finds that the proper time increment on the moving clock is approximately given by

$$d\tau = ds/c = \left( 1 + \frac{V - \Phi_0}{c^2} - \frac{v^2}{2c^2} \right) dt. \quad (5)$$

Considering an elliptic orbit with semi-major axis  $a$ , and taking  $V = V_N \approx GM_E/r$ , this may be recast into the form [1]

$$d\tau = ds/c = \left[ 1 + \frac{3GM_E}{2ac^2} + \frac{\Phi_0}{c^2} - \frac{2GM_E}{c^2} \left( \frac{1}{a} - \frac{1}{r} \right) \right] dt. \quad (6)$$

The first constant rate correction terms in the above amount to

$$\frac{3GM_E}{2ac^2} + \frac{\Phi_0}{c^2} = -4.7454 \times 10^{-10}, \quad (7)$$

for the Galileo system, and  $-4.4647 \times 10^{-10}$ , for the GPS system; this indicates that the orbiting clock is beating faster, by about  $41 \mu\text{s/day}$ , for the Galileo system, and  $39 \mu\text{s/day}$ , for the GPS system. For this reason, the GPS system has a built in frequency offset of this magnitude, while the increased computational capabilities made available to current and future receivers of the Galileo system leave this correction to the user. The residual periodic corrections, proportional to  $1/r - 1/a$ , have an amplitude of order  $49 \text{ ns/day}$ , for the Galileo system, and  $46 \text{ ns/day}$ , for the GPS system.

### C. Shapiro time delay and the Sagnac effect

The so-called Shapiro time delay, a second order relativistic effect due to the signal propagation is given by [1]

$$\Delta t_{\text{delay}} = \frac{\Phi_0 l}{c^3} + \frac{2GM_E}{c^3} \ln \left( 1 + \frac{l}{R_E} \right) , \quad (8)$$

where we have integrated over a straight line path of (proper) length  $l$ . Evaluating this delay, one concludes that this effect amounts to  $6.67 \times 10^{-11}$  s.

Also, one must consider the so-called Sagnac effect, which arises from the difference between the gravitational potential  $V$  and the effective potential  $\Phi$ , when proceeding from a non-rotational to a rotational frame. Hence, one gets the additional time delay

$$\Delta t_{\text{Sagnac}} = \frac{\omega}{c^2} \int_{\text{path}} r^2 d\phi = \frac{2\omega}{c^2} \int_{\text{path}} dA_z , \quad (9)$$

where  $dA_z$  is the orto-equatorial projection of the area element swept by a vector from the rotation axis to the satellite. For the Galileo system, this yields a maximum value of 153 ns while, for the GPS system, one gets 133 ns.

One concludes this section by recalling the main effects affecting the considered global positioning systems: a frequency shift of order  $10^{-10}$  and a propagation time delay (Shapiro plus Sagnac effect) of the order  $10^{-7}$  s. In what follows, one shall compute the additional frequency shift and propagation time delay induced by common proposals for extensions of GR, and compare the results with the above quantities, plus the frequency accuracy of  $10^{-12}$  and the time accuracy of Galileo, of order  $10^{-9}$  s, which corresponds to a optimistic spatial accuracy of 30 cm.

### D. Post-Newtonian effects

We address now the issue of measuring Post-Newtonian effects with the Galileo positioning system. As the moniker indicates, these are effects below the Newtonian order, that is,  $GM_E/R_E c^2 \approx 10^{-10}$ . A general formalism exists to describe lower-order effects induced by extensions to GR and alternate theories that resort to a metric approach of gravity; indeed, any such theory may be analysed locally and compared with the so-called Parameterised Post-Newtonian (PPN) metric [6, 7], given by the line element

$$ds^2 = - \left[ 1 - \frac{2V}{c^2} + 2\beta \left( \frac{V}{c^2} \right)^2 \right] (c dt)^2 + \left( 1 - 2\gamma \frac{V}{c^2} \right) dV . \quad (10)$$

In the above, the parameter  $\beta$  measures the non-linearity of the superposition law for gravity, while  $\gamma$  indicates the space curvature produced per unit mass. For clarity, we consider only a simplified version of the full PPN metric; the latter encompasses ten PPN parameters, characterising the underlying fundamental theory, and may be related to violation of momentum conservation, existence of a privileged reference frame, amongst others. GR is characterised by  $\beta = \gamma = 1$ , while all remaining parameters vanish. For that reason, the quantities  $\beta - 1$  and  $\gamma - 1$  measure the deviation from the predictions of the currently accepted theory. Experimentally, it is found that  $|\beta - 1| \leq 2 - 3 \times 10^{-4}$  (Nordtvedt effect) and  $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$  (Cassini radiometry).

Unfortunately, it is clear from the above equation that Post-Newtonian effects arise only at an order  $\sim 10^{-20}$ , undetectable by the accuracy of the GPS and Galileo systems.

## III. DETECTION OF THE COSMOLOGICAL CONSTANT

Latest observations indicate that the Universe is experiencing an accelerated expansion, which may be characterised by the presence of a cosmological constant  $\Lambda \sim 10^{-35} \text{ s}^{-2}$ , acting as a negative-pressure fluid (see *e.g.* [8] and references therein). By matching the outer Friedmann-Robertson-Walker metric with a static, symmetric solution given by Birkhoff's theorem, we may derive the Schwarzschild-de Sitter metric, given by the line element (in anisotropic form) [9],

$$ds^2 = - \left( 1 - \frac{2V_N}{c^2} - \frac{\Lambda r^2}{3c^2} \right) (c dt)^2 + \frac{1}{1 - \frac{2V_N}{c^2} - \frac{\Lambda r^2}{3c^2}} dr^2 + d\Omega \quad . \quad (11)$$

This indicates that the cosmological constant induces an additional term to the potential, of the form  $V_\Lambda = -\Lambda r^2/6$ ; since its expected effect is assumed to be small, one may neglect the issue of performing a coordinate change to an isotropic, co-rotating frame of reference, as well as the identification of proper time with clocks at rest on the surface of the geoid (however, notice that the identification of proper time as that measured by a clock at rest at infinity breaks down, due to the Schwarzschild “bubble” breaking down at a distance  $r$  given by the condition  $V_n = V_\Lambda$ ).

The frequency shift of a signal emitted at a distance from the origin  $r = R_E + h$  (for the Galileo system,  $h = 17.2 \times 10^6$  m) and received at a distance  $r = R_E$  is given by

$$\left( \frac{f_{Earth}}{f_{Sat}} \right) = \sqrt{\frac{g_{00 Earth}}{g_{00 Sat}}} = \sqrt{\frac{1 - 2V(R_E)/c^2}{1 - 2V(R_E + h)/c^2}} \simeq \frac{V(R_E) - V(R_E + h)}{c^2} \quad . \quad (12)$$

Hence, one may compute the additional frequency shift induced by this extra potential contribution, through

$$\left( \frac{f_{Earth}}{f_{Sat}} \right)_\Lambda \simeq \frac{V_\Lambda(R_E) - V_\Lambda(R_E + h)}{c^2} = \frac{\Lambda}{6c^2} h(2R_E + h) \sim 10^{-38} \quad , \quad (13)$$

which clearly falls below the accuracy  $\epsilon_{f_r} = 10^{-12}$  of the Galileo constellation.

Likewise, the propagational time delay is given by

$$\Delta t_{delay} = \frac{1}{c} \int_{R_E}^{R_E + h} V(r) dr \quad . \quad (14)$$

Hence, the cosmological constant induces a further delay of

$$\Delta t_\Lambda = \frac{1}{c} \int_{R_E}^{R_E + h} \frac{\Lambda r^2}{6c^2} dr = \frac{\Lambda}{18c^3} h [(3R_E(R_E + h) + h^2)] \sim 10^{-40} \text{ s} \quad , \quad (15)$$

also many orders of magnitude below the time resolution of  $10^{-9}$  s. Therefore, one concludes that the cosmological constant is completely undetectable by the Galileo system.

#### IV. DETECTION OF ANOMALOUS, CONSTANT ACCELERATION

An anomalous constant acceleration could model first-order effects arising from some fundamental theory of gravitation which expands upon GR, or indicate some threshold between known dynamics and yet undetected, exotic physics. One examples stems from the so-called Modified Newtonian Dynamics (MOND) model [10–12], which attempts to account for the missing matter problem indicated by galactic rotation curves without the need for dark matter, by featuring a departure from the classical Poisson equation at a characteristic acceleration scale of  $10^{-10} \text{ m/s}^2$ . Also, although yet unmodelled or theoretically unaccounted for, an anomalous, sunbound, constant acceleration  $a = (8.74 \pm 1.33) \times 10^{-10} \text{ m/s}^2$  has been reported to affect the Pioneer 10/11 probes [13–15].

An anomalous, constant acceleration  $a$  may be phenomenologically modelled by a potential  $V_a = ar$ ; following the procedure depicted in the previous section, the following frequency shift is obtained

$$\left( \frac{f_{Earth}}{f_{Sat}} \right)_a \simeq \frac{V_a(R_E) - V_a(R_E + h)}{c^2} = \frac{ah}{c^2} \quad . \quad (16)$$

Comparing with the frequency accuracy  $\epsilon_{f_r} = 10^{-12}$ , one finds that only accelerations  $a \geq c^2 \epsilon_{f_r} / h \sim 10^{-3} \text{ m/s}^2$  may be detected.

The propagational time delay due to this extra potential addition is given by

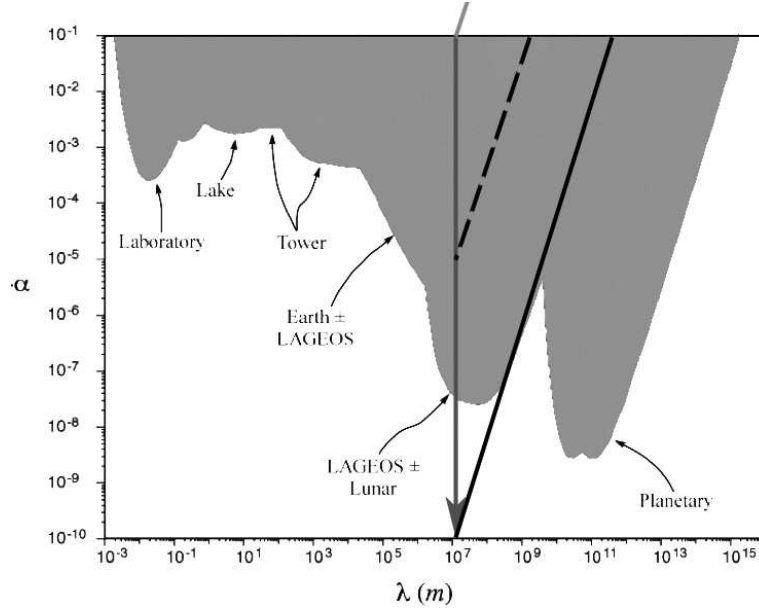


FIG. 1: Exclusion plot for the Yukawa strength  $\alpha$  and range  $\lambda$ , and superimposed limits obtained for varying frequency accuracy  $\epsilon_{fr}$ :  $10^{-10}$  (grey, full),  $10^{-12}$  (black dash) and  $10^{-19}$  (black full).

$$\Delta t_a = \frac{1}{c} \int_{R_E}^{R_E+h} \frac{ar}{c^2} dr = \frac{a}{2c^3} h(2R_E + h) , \quad (17)$$

and comparison with a time accuracy of  $10^{-9}$  s yields the condition for detectability  $a \gtrsim 100$  m/s<sup>2</sup>. Therefore, one concludes that accelerations of the order  $10^{-10} - 10^{-9}$  m/s<sup>2</sup> are beyond the observable reach of the Galileo system; conversely, detectability of a constant acceleration of the order of  $10^{-10}$  m/s<sup>2</sup> would require an improvement of 7 orders of magnitude in frequency accuracy (to  $\epsilon_{fr} \sim 10^{-19}$ ) and 12 orders of magnitude in time resolution (to  $10^{-21}$  s).

## V. DETECTION OF YUKAWA POTENTIAL

A common phenomenological approach to extensions of GR lies in assuming that the potential has a modified Yukawa form,

$$V(r) = -\frac{G_\infty M_E}{r} \left( 1 + \alpha e^{-r/\lambda} \right) , \quad (18)$$

where  $\alpha$  is the strength of the perturbation,  $\lambda$  its characteristic range, and  $G_\infty$  the gravitational coupling for  $r \rightarrow \infty$ ; the latter may be regarded as a redefinition of Newton's constant  $G$ , through  $G = G_\infty(1 + \alpha)$ . This potential may be separated into a Newtonian-like potential and an extra potential  $V_Y = -(\alpha G M_E / (1 + \alpha) r) e^{-r/\lambda}$ . The Yukawa contribution may arise from scalar/tensor field models, where the range is related to the mass  $m$  of the scalar field,  $\lambda \propto m^{-1}$  [5].

Tight experimental constraints are available, stemming from several sources and regimes, as may be seen in Fig. 1. Clearly, two yet unexplored avenues remain: the sub-millimeter regime,  $\lambda < 10^{-3}$  m [16], and an astronomical regime,  $\lambda > 10^{15}$  m  $\approx 0.1$  ly.

Following the previous steps, one first obtains the extra frequency shift

$$\left( \frac{f_{Earth}}{f_{Sat}} \right)_Y = \frac{V_Y(R_E) - V_Y(R_E + h)}{c^2} = \frac{G M_E}{c^2 R_E} \left( \frac{\alpha}{1 + \alpha} \right) e^{-R_E/\lambda} \left( e^{-h/\lambda} \frac{R_E}{R_E + h} - 1 \right) . \quad (19)$$

The additional time delay is given by

$$\Delta t_Y = \frac{1}{c} \int_{R_E}^{R_E+h} \frac{GM_E}{c^2 r} \left( \frac{\alpha}{1+\alpha} \right) e^{-r/\lambda} dr . \quad (20)$$

The above expressions may be considerably shortened if it is assumed that this additional “fifth-force” is a long-range,  $\lambda \gg r$ , or short-range interaction,  $\lambda \ll r$ .

### A. Short-range fifth force

If the range of the Yukawa interaction is short-ranged,  $\lambda \ll h, R_E$ , one obtains

$$\left( \frac{f_{Earth}}{f_{Sat}} \right)_Y \simeq -\frac{GM_E}{c^2 R_E} \left( \frac{\alpha}{1+\alpha} \right) e^{-R_E/\lambda} . \quad (21)$$

If this effect is undetectable within the frequency accuracy  $\epsilon_{f_r}$ , one obtains the constraint for small  $\alpha$

$$\alpha \lesssim \left[ \frac{GM_E}{c^2 R_E} \right]^{-1} e^{R_E/\lambda} \epsilon_{f_r} \approx 1.4 \times 10^{-3} e^{R_E/\lambda} \gg 1 , \quad (22)$$

which yields no new insight into the yet unexplored sub-millimetric regime, as depicted in Fig. 1.

Likewise, the additional propagation time delay is given by

$$\Delta t_Y = -\frac{GM_E \alpha}{c^3} \ln \left( 1 + \frac{h}{R_E} \right) , \quad (23)$$

so that comparison with the time accuracy of  $\Delta t = 10^{-9}$  yields, for  $\alpha \ll 1$

$$\alpha \leq \left[ \frac{GM_E}{c^3} \ln \left( 1 + \frac{h}{R_E} \right) \right]^{-1} \Delta t \approx 50 . \quad (24)$$

Hence, one concludes that the short-range regime of a hypothetical Yukawa fifth force cannot be probed by the Galileo system.

### B. Long-range fifth force

If one follows the inverse assumption of the previous subsection, and assumes a long range fifth force,  $\lambda \gg h, R_E$ , the exponential terms may be expanded to first order in  $r/\lambda$ ; as a result, the induced propagation time delay becomes

$$\Delta t_Y \simeq -\frac{GM_E \alpha}{c^3} \frac{h}{\lambda} . \quad (25)$$

If the effect is undetected at a level of accuracy  $\Delta t \sim 10^{-9}$  s, one obtains, for small  $\alpha$

$$|\alpha| < \frac{c^3 \Delta t}{GM_E} \frac{\lambda}{h} \approx 4 \times 10^{-6} \left( \frac{\lambda}{1 \text{ m}} \right) . \quad (26)$$

For a lower bound of  $\lambda \approx 10^8 \text{ m}$  (only one order of magnitude above  $R_E, h$ ), we obtain the incompatible result  $\alpha < 400$ .

Regarding the additional frequency shift, one finds

$$\left( \frac{f_{Earth}}{f_{Sat}} \right)_Y \simeq \frac{GM_E \alpha h}{2c^2 \lambda^2} , \quad (27)$$

so that comparison with the frequency accuracy level of  $\epsilon_{f_r} \sim 10^{-12}$  yields, for  $\alpha \ll 1$

$$\alpha < \left( \frac{GM_E}{c^2} \right)^{-1} \left( \frac{2\lambda^2}{h} \right) \epsilon_{f_r} \approx 10^{-5} \epsilon_{f_r} \left( \frac{\lambda}{1 \text{ m}} \right)^2, \quad (28)$$

or, equivalently, a quite interesting bound

$$\log \alpha < -5 + \log \epsilon_{f_r} + 2 \log \left( \frac{\lambda}{1 \text{ m}} \right). \quad (29)$$

One may plot the different constraints obtained by varying the frequency accuracy  $\epsilon_{f_r}$ , as seen in Fig. 1; this shows that, at the current level, no new bounds are produced. Also, it shows that, at a level  $\epsilon_{f_r} \sim 10^{-19}$ , the region below the “trough” at  $\lambda \sim 10^8 \text{ m}$  (corresponding to  $\alpha < 10^{-8}$ ) could be investigated.

## VI. CONCLUSIONS

In this work, we have addressed the possibility of detecting signals of new physics through the use of the Galileo positioning system. This application could be valuable, as any unexpected new phenomenology could provide further insight into what lies beyond General Relativity. We have specifically looked at the propagation time delay and frequency shift induced by three different models, namely a potential related to the presence of the cosmological constant, the influence of an anomalous, constant acceleration, and the addition of a Yukawa-like fifth force. We also briefly discussed the (im)possibility of measuring post-Newtonian effects with the Galileo system.

Unfortunately, our conclusions indicate that the available observables are not suitable for the intended purpose; indeed, while these render the detection of the cosmological constant unattainable, they also indicate that the current accuracy is many orders of magnitude above that needed to probe interesting regimes of anomalous constant acceleration ( $a \sim 10^{-10} - 10^{-9} \text{ m/s}^2$ ) or Yukawa range  $\lambda > 10^8 \text{ m}$  and strength  $\alpha < 10^{-8}$ . Indeed, a frequency accuracy of  $10^{-19}$ , near the “quantum” regime, is required to further probe the desired scales. Although this is clearly beyond the grasp of any foreseeable global positioning systems, and yet unavailable in space certified clocks, such precision might be attainable in the future.

Finally, we remark that, although it was not the purpose of this study, the Galileo positioning system could be paramount in improving the bound on violation of the Local Positioning Invariance (LPI) principle [5]; this tenant, one of the fundamental pillars of General Relativity, postulates that clock rates are independent of their spacetime positions. Experimental constraints on allowed relative frequency deviations indicate that this invariance holds down to a level of  $2.1 \times 10^{-5}$  [17]. Endowing one or more elements of the Galileo constellation with higher precision clocks and allowing for sufficiently stable communication with stations on Earth, possibly through a microwave link, could yield an improvement of up to two orders of magnitude on the LPI. Another alternative could involve installing cornercubes on the surface of one or more elements of the Galileo system, so to allow for accurate laser ranging. It is tempting to call this subset of the Galileo constellation *Siderius Nuncius*, the Celestial Messenger, given its potential in helping to sort out the mysteries of the Cosmos.

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